CAREER SPECTRA DEHRADUN

PHYSICS ACADEMY

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Dedicated to excellence...

GATE

(MATHEMATICAL PHYSICS)

PREVIOUS YEAR'S QUESTIONS WITH ANSWER (CHAPTER-WISE)

- MATRICES
- VECTOR ANALYSIS
- FOURIER ANALYSIS & LAPLACE TRANSFORM
- COMPLEX ANALYSIS
- DIFFERENTIAL EQUATION
- OTHER QUESTIONS

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MATRICES

1. Consider an anti-symmetric tensor P_{ij} with indices I and j running from 1 to 5. The number of independent components of the tensor is. **[GATE-2010]**

(a) 3 (b) 10 (c) 9 (d) 6

2. The eigenvalues of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are. **[GATE-2010]**

(a) 5, 2, -2 (b) -5, -1, -1

(c) 5, 1, -1 (d) -5, 1, 1

3. Two matrices *A* and *B* are said to be similar if $B= P^{-1}AP$ for some invertible matrix P. Which of the following statements is NOT TRUE? **[GATE-2010]**

(a) Det *A* = Det *B*

(b) Trace of *A* = Trace of *B*

(c) *A* and *B* have the same eigenvectors

(d) *A* and *B* have the same eigenvalues

4. A 3×3 matrix has elements such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalues of the matrix is. **[GATE-2010]**

(a) 18 (b) 12 (c) 9 (d) 6

5. The number of independent components of the symmetric tensor A_{ij} with indices $i, j = 1,2,3$ is. **[GATE-2012]**

(a) 1 (b) 3 (c) 6 (d) 9

6. The eigenvalue of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ are **[GATE-2012]**

(a) 0, 1, 1 (b) $0, -\sqrt{2}, \sqrt{2}$ (c) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (d) $\sqrt{2}, \sqrt{2}, 0$

7. The degenerate eigenvalue of the matrix $\begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}$ is (your answer should be an integer) ____________ **[GATE-2013]**

8. The matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is. **[GATE-2014]**

(a) Orthogonal (b) symmetric

(c) anti-symmetric (d) Unitary

9. Let X be a column vector of dimension n > 1with at least one non-zero entry. The number of non-zero eigenvalues of the matrix $M = XX^T$ is. **[GATE-2017]**

(a) 0 (b) n (c) 1 (d) n -1

10. The eigenvalues of a Hermitian matrix are all. **[GATE-2018]**

(a) Real (b) Imaginary

(c) Of modulus one (d) Real and positive

11. During a rotation , along the axis of rotation remain unchanged. For the rotation matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$, the vector along the axis of rotation is. **[GATE 2019]**

(a) $\frac{1}{3}(2\hat{\imath} - j + 2\hat{k})$ (b) $\frac{1}{\sqrt{3}}(\hat{\imath} + j - \hat{k})$

(c) $\frac{1}{\sqrt{3}}(\hat{\imath} - j - \hat{k})$ (d) $\frac{1}{3}(2\hat{\imath} + 2j - \hat{k})$

12. A real, invertible 3 × 3 matrix M has eigenvalues λ_i,(i = 1,2,3) and the corresponding eigenvectors are $|e_i\rangle$,(i = 1,2,3) respectively. Which one of the following is correct? **[GATE 2020]**

(a) $M|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$, for i = 1, 2, 3

(b) $M^{-1}|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$, for i = 1,2,3

(c) $M^{-1}|e_i\rangle = \lambda_i|e_i\rangle$, for i = 1, 2, 3

(d) The eigenvalues of M and M^{-1} are not related

13. The product of the eigenvalues of $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is. **[GATE 2020]**

(a) -1 (b) 1 (c) 0 (d) 2

14. Let $|e_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, |e_2\rangle = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, |e_3\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Let S= $|e_1\rangle|e_1\rangle|e_1\rangle$ $\{|e_1\rangle, |e_1\rangle, |e_1\rangle\}$. Let $\mathbb{R}^3$ denote the three dimensional real vector space. Which one of the following is correct? **[GATE 2020]**

(a) S is an orthogonal set

(b) S is a linearly dependent set

(c) S is the basis of $\mathbb{R}^3$

(d) $\sum_{i=1}^{3}|e_i\rangle\langle e_i| = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

VECTOR ANALYSIS

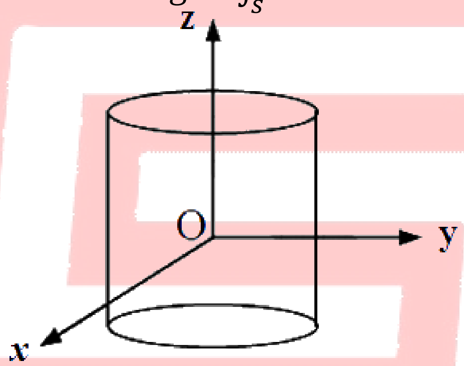
1. If a force $\vec{F}$ is derivable from a potential function $V(r)$, where r is the distance from the origin of the coordinate system, it follows that: **[GATE-2011]**

(a) $\vec{\nabla} \times \vec{F} = 0$ (b) $\vec{\nabla}.\vec{F} = 0$ (c) $\vec{\nabla}V = 0$ (d) $\nabla^2 V = 0$

2. The unit vector normal to the surface $x^2 + y^2 - z = 1$ at the point $P(1, 1, 1)$ is: : **[GATE-2011]**

(a) $\frac{\hat{\imath}+\hat{\jmath}-\hat{k}}{\sqrt{3}}$ (b) $\frac{2\hat{\imath}+\hat{\jmath}-\hat{k}}{\sqrt{6}}$ (c) $\frac{\hat{\imath}+2\hat{\jmath}-\hat{k}}{\sqrt{6}}$ (d) $\frac{2\hat{\imath}+2\hat{\jmath}-\hat{k}}{3}$

3. Consider a cylinder of height h and radius a, closed at both ends, centered at the origin. Let $\vec{r} = \hat{\imath}x + \hat{\jmath}y + \hat{k}z$ be the position vector and $\hat{n}$ be a unit vector normal to the surface. The surface integral $\int_s \vec{r}.\hat{n}ds$ over the closed surface of the cylinder is: **[GATE-2011]**



(a) $2\pi a^2(a + h)$ (b) $3\pi a^2 h$

(c) $2\pi a^2 h$ (d) zero

4. Identify the correct statement for the following vectors $\vec{a} = 3\hat{\imath} + 2\hat{\jmath}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath}$. **[GATE-2012]**

(a) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent

(b) The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent

(c) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal

(d) The vectors $\vec{a}$ and $\vec{b}$ are normalized

5. If $\vec{A}$ and $\vec{B}$ are constant vectors, then $\vec{\nabla}(\vec{A}.(\vec{B} \times \vec{r}))$ is. **[GATE-2013]**

(a) $\vec{A}.\vec{B}$ (b) $\vec{A} \times \vec{B}$ (c) $\vec{r}$ (d) zero

6. The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point (1, 1, 1) is: **[GATE-2014]**

(a) $\frac{\hat{x}+\hat{y}-\hat{z}}{\sqrt{3}}$ (b) $\frac{\hat{x}-\hat{y}-\hat{z}}{\sqrt{3}}$ (c) $\frac{\hat{x}-\hat{y}+\hat{z}}{\sqrt{3}}$ (d) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$

7. The direction of $\vec{\nabla} f$ for a scalar field $f(x,y,z)=\frac{1}{2}x^2-xy+\frac{1}{2}z^2$ at the point P(1,1,2) is. **[GATE-2016]**

(a) $\frac{(-\hat{j}-2\hat{k})}{\sqrt{5}}$ (b) $\frac{(-\hat{j}+2\hat{k})}{\sqrt{5}}$ (c) $\frac{(\hat{j}-2\hat{k})}{\sqrt{5}}$ (d) $\frac{(\hat{j}+2\hat{k})}{\sqrt{5}}$

8. In spherical polar coordinates (r,θ,ϕ), the unit vector $\hat{\theta}$ at $(10,\pi/4,\pi/2)$ is. **[GATE-2018]**

(a) $\hat{k}$ (b) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$ (c) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (d) $\frac{1}{\sqrt{2}}(\hat{j}-\hat{k})$

9. Given $\vec{V}_1=\hat{\imath}-\hat{j}$ and $\vec{V}_2=-2\hat{\imath}+3\hat{j}+2\hat{k}$, which one of the following $\vec{V}_3$ makes $(\vec{V}_1,\vec{V}_2,\vec{V}_3)$ a complete set for a three dimensional real linear vector space? **[GATE-2018]**

(a) $\vec{V}_3=\hat{\imath}+\hat{j}+4\hat{k}$ (b) $\vec{V}_3=2\hat{\imath}-\hat{j}+2\hat{k}$
(c) $\vec{V}_3=\hat{\imath}+2\hat{j}+6\hat{k}$ (d) $\vec{V}_3=2\hat{\imath}+\hat{j}+4\hat{k}$

FOURIER ANALYSIS & LAPLACE TRANSFORM

1. If $f(x)=\begin{cases}0 & for\ x<3\\ x-3 & for\geq 3\end{cases}$ then the Laplace transform f(x) is. **[GATE-2010]**

(a) $s^{-2}e^{3s}$ (b) s^2e^{3s} (c) s^{-2} (d) $s^{-2}e^{-3s}$

2. The value of $\int_0^3 t^2\delta(3t-6)dt$ ……………………….(upto one decimal place) **[GATE-2015]**

3. The Heaviside function is defined as H(t) = $\begin{cases}+1, & for\ t>0\\ -1, & for\ t<0\end{cases}$ and its Fourier transform is given by $-\frac{2i}{\omega}$. The Fourier transform of $\frac{1}{2}$ [H(t+1/2)-H(t-1/2)] is. **[GATE-2015]**

(a) $\frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}}$ (b) $\frac{\cos\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}}$ (c) $\sin\left(\frac{\omega}{2}\right)$ (d) 0

4. A periodic function *f(x)* of period 2π is defined in the interval $(-\pi<x<\pi)$ **[GATE-2016]**

$$f(x)=\begin{cases}-1, & -\pi<x<0\\ 1, & 0<x<\pi\end{cases}$$

(a) The appropriate Fourier series expansion for f(x)is.
(b) $f(x)=\left(\frac{4}{\pi}\right)[\sin x+\frac{\sin 3x}{3}+\frac{\sin 5x}{5}+\cdots]$
(c) $f(x)=\left(\frac{4}{\pi}\right)[\sin x-\frac{\sin 3x}{3}+\frac{\sin 5x}{5}-\cdots]$

(d) $f(x) = \left(\frac{4}{\pi}\right)[\cos x + \frac{\cos 3x}{3} + \frac{\cos 5x}{5} + \cdots]$

(e) $f(x) = \left(\frac{4}{\pi}\right)[\cos x - \frac{\cos 3x}{3} + \frac{\cos 5x}{5} - \cdots]$

5. The coefficient of e^{ikx} in the Fourier expansion of $u(x) = A sin^2(\alpha x)$ for $k = -2\alpha$ is. **[GATE-2017]**

(a) A/4 (b) –A/4 (c) A/2 (d) –A/2

6. Let θ be a variable in the range $-\pi \le \theta < \pi$. Now consider a function

$$\psi(\theta) = \begin{cases} 1 & for \frac{-\pi}{2} \le \theta < \frac{\pi}{2} \\ 0 & otherwise \end{cases}$$ **[GATE-2019]**

if its Fourier-series is written as $\psi(\theta) = \sum_{m}^{\infty} = -\infty C_m e^{-im\theta}$, then the value of $|C_3|^2$ (rounded off to three decimal places) is__________

7. If x= $\sum_{k=1}^{\infty} a_k sinkx$, for –л ≤ x ≤ л, the value of a_2 is ______. **[GATE-2020]**

8. Let f_n (x) = $\begin{cases} 0, & x < -\frac{1}{2n} \\ n, & \frac{-1}{2n} < x < \frac{1}{2n} \\ 0, & \frac{1}{2n} < x \end{cases}$ The value of $\lim_{n\to\infty} \int_{-\infty}^{\infty} f_n(x) sinxdx$ is____.

[GATE-2020]

COMPLEX ANALYSIS

1. The value of the integral $\oint_C \frac{e^z \sin(z)}{z^2} dz$, where the contour C is the unit circle: |z-2|=1, is **[GATE-2010]**

(a) $2\pi i$ (b) $4\pi i$ (c) πi (d) 0

2. Which of the following statements is **TRUE** for the function $f(z) = \frac{z \sin z}{(z-\pi)^2}$? **[GATE-2010]**

(a) f(z) is analytic everywhere in the complex plane

(b) f(z) has a zero at $z = \pi$

(c) f(z) has a pole of order 2 at $z = \pi$

(d) f(z) has a simple pole at $z = \pi$

3. Consider a counterclockwise circular contour |z| = 1 about the origin. Let f(z) = $\frac{z \sin z}{(z-\pi)^2}$. then the integral $\oint f(dz)$ over this contour is: **[GATE-2011]**

(a) $-i\pi$ (b) zero (c) $i\pi$ (d) $2i\pi$

4. For the function $f(z) = \frac{16z}{(z+3)(z-1)^2}$, residue at the pole $z = 1$ is (your answer should be an integer) _____________. **[GATE-2013]**

5. The value of the integral $\oint_C \frac{z^2}{e^z+1} dz$ where C is the circle |z| = 4, is. **[GATE-2014]**

(a) $2\pi i$ (b) $2\pi^2 i$ (c) $4\pi^3 i$ (d) $4\pi^2 i$

6. Consider a complex function $f(z) = \frac{1}{z\left(z+\frac{1}{2}\right)\cos(z\pi)}$. Which one of the following statements is correct? **[GATE-2015]**

(a) f (z) has simple poles at z = 0 and z = $-\frac{1}{2}$

(b) f (z) has second order pole at z = = $-\frac{1}{2}$

(c) f (z) has infinite number of second order poles

7. If $f(x) = e^{-x^2}$ and $g(x) = |x|e^{-x^2}$, then. **[GATE-2015]**

(a) f and g are differentiable everywhere

(b) f is differentiable everywhere but g is not

(c) g is differentiable everywhere but f is not

(d) g is discontinuous at $x = 0$

8. Consider $w = f(z) = u(x,y) + iv(x,y)$ to be an analytic function in a domain D . Which one of the following options is NOT correct? **[GATE-2014]**

(a) u(x,y) satisfies Laplace equation in D

(b) v (x,y) satisfies Laplace equation inD

(c) $\int_{z_1}^{z_2} f(z)dz$ is dependent on the choice of the contour between z_1 and z_2 in D

(d) f(z) can be Taylor expended in D

9. Which of the following is an analytic function of z everywhere in the complex plane? **[GATE-2016]**

(a) z^2 (b) $(z^*)^2$ (c) $|z|^2$ (d) $\sqrt{z}$

10. The contour integral $\oint \frac{dz}{1+z^2}$ evaluated along a contour going from $-\infty$ to $+\infty$ along the real axis and closed in the lower half-plane circle is equal to………….. (up to two decimal places). **[GATE-2017]**

11. The imaginary part of an analytic complex function is $v(x,y)=2xy+3y$.The real part of the function is zero at the origin. The value of the real part of the function at 1+ i is …………….. (up to two decimal places) **[GATE-2017]**

12. The absolute value of the integral $\int \frac{5z^3+3z^2}{z^2-4} dz$, over the circle |z-1.5|=1 in complex plane, is __________ (up to two decimal places). **[GATE-2018]**

13. The pole of the function $f(z) = \cot z$ at z = 0 is. **[GATE-2019]**
(a) a removable pole
(b) an essential singularity
(c) a simple pole
(d) a second order pole

14. The value of the integral $\int_{-\infty}^{\infty} \frac{\cos(kx)}{x^2+a^2} dx$, where k > 0 and a > 0, is **[GATE-2019]**
(a) $\frac{\pi}{a} e^{-ka}$
(b) $\frac{2\pi}{a} e^{-ka}$
(c) $\frac{\pi}{2a} e^{-ka}$
(d) $\frac{3\pi}{2a} e^{-ka}$

15. For a complex variable z and the contour c: |z|=1 taken in the counter clockwise direction $\frac{1}{2\pi i} \oint_c \left(z - \frac{2}{z} + \frac{3}{z^2}\right) dz =$______. **[GATE-2020]**

DIFFERENTIAL EQUATION

1. The solution of the differential equation for y(t) : $\frac{d^2y}{dt^2} - y = 2\cosh(t)$, subject to the initial conditions y(0) = 0 and $\frac{dy}{dt}|_{t=0} = 0$, is. **[GATE-2010]**
(a) $\frac{1}{2}\cos h(t) + t\sinh(t)$
(b) $-\sinh(t) + \cos h(t)$
(c) t cosh(t)
(d) t sinh(t)

2. The solutions to the differential equation $\frac{dy}{dx} = -\frac{x}{y+1}$ are a family of. **[GATE-2010]**
(a) Circles with different radii
(b) Circles with different centres
(c) Straight lines with different slopes
(d) Straight lines with different intercepts on the *y*-axis

3. The solution of the differential equation $\frac{d^2y}{dt^2} - y = 0$, subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$ is. **[GATE-2014]**
(a) cos t + sin t
(b) cosht + sinh t
(c) cos t- sin t
(d) cosh t- sinh t

4. A function y(z) satisfies the ordinary differential equation $y" + \frac{1}{2}y' - \frac{m^2}{z^2}y = 0$, where *m* = 0, 1, 2, 3,..... Consider the four statements P, Q, R, S as given below.
P: z^m and z^{-m} are linearly independent solutions for all values of *m*
Q: z^m and z^{-m} are linearly independent solutions for all values of *m* > 0
R: ln *z* and 1 are linearly independent solutions for *m* = 0

S: z^m and ln z are linearly independent solutions for all values of m
The correct option for the combination of valid statements is

[GATE-2015]

(a) P, R and S only (b) P and R only
(c) Q and R only (d) R and S only

5. Consider the linear differential equation $\frac{dy}{dx} = xy$. If y = 2 at x = 0, the value of y at x = 2 is given by. **[GATE-2016]**

(a) e^{-2} (b) $2e^{-2}$ (c) e^{-2} (d) $2e^{2}$

6. Consider the differential equation $\frac{dy}{dx} + y\tan(x) = \cos(x)$. If y(0) = 0, y $\left(\frac{\pi}{3}\right)$ is …………... (up to two decimal places). **[GATE-2017]**

7. Given $\frac{d^2f(x)}{dx^2} - 2\frac{df(x)}{dx} + f(x) = 0$ and boundary conditions f(0)=1 and f1(0)=0, the value of f(0.5) is __________ (up to two decimal places). **[GATE-2018]**

8. For the differential equation $\frac{d^2y}{dx^2} - n(n+1)\frac{y}{x^2} = 0$, where n is a constant, the product of its two independent solutions is. **[GATE-2019]**

(a) $\frac{1}{x}$ (b) x (c) x^n (d) $\frac{1}{x^{n+1}}$

9. Which one of the following is a solution of $\frac{d^2u(x)}{dx^2} = k^2u(x)$, for k real ?

[GATE-2020]

(a) e^{-kx} (b) sin kx (c) cos kx (d) sinh x

OTHER QUESTIONS

1. The number of distinct ways of placing four indistinguishable balls into five distinguishable boxes is ___________. **[GATE-2013]**

2. The scale factors corresponding to the covariant metric tensor g_{ij} in spherical polar coordinates are. **[GATE-2018]**

(a) 1, r^2, $r^2 \sin^2\theta$ (b) 1, $r^2 \sin^2\theta$
(c) 1, 1, 1 (d) 1, r, r, $\sin\theta$

ANSWER KEY

MATRICES

1.	B	**2.**	C	**3.**	C	**4.**	D	**5.**	C	**6.**	B
7.	2, 5, 5	**8.**	D	**9.**	C	**10.**	A	**11.**	B	**12.**	A
13.	C	**14.**	C								

VECTOR ANALYSIS

1.	A	**2.**	D	**3.**	B	**4.**	A	**5.**	D	**6.**	D
7.	B	**8.**	D	**9.**	D						

FOURIER ANALYSIS & LAPLACE TRANSFORM

1.	D	**2.**	1.33	**3.**	A	**4.**	A	**5.**	B	**6.**	0.010 to 0.013
7.	1	**8.**	0								

COMPLEX ANALYSIS

1.	D	**2.**	C	**3.**	B	**4.**	3	**5.**	C	**6.**	D
7.	B	**8.**	C	**9.**	A	**10.**	π	**11.**	3	**12.**	81.64
13.	C	**14.**	A	**15.**	-2						

DIFFERENTIAL EQUATION

1.	D	**2.**	A	**3.**	D	**4.**	C	**5.**	D	**6.**	0.52
7.	0.81	**8.**	B	**9.**	A						

OTHER QUESTIONS

1.	120	**2.**	D	